Nontrivial temperature behavior of the carrier concentration in graphene on ferroelectric substrate with domain walls

Anatolii I. Kurchak1, Anna N. Morozovska2,3, Eugene A. Eliseev4, Sergei V. Kalinin5 and Maksym V. Strikha1,6

1 V.Lashkariov Institute of Semiconductor Physics, National Academy of Sciences of Ukraine, pr. Nauky 41, 03028 Kyiv, Ukraine
2 Institute of Physics of National Academy of Sciences of Ukraine, pr. Nauky 46, 03028 Kyiv, Ukraine
3 Bogolyubov Institute for Theoretical Physics, National Academy of Sciences of Ukraine, 14-b Metrolohichna str. 03680 Kyiv, Ukraine
4 Institute for Problems of Materials Science, National Academy of Sciences of Ukraine, Krijianovskogo 3, 03142 Kyiv, Ukraine
5 The Center for Nanophase Materials Sciences, Oak Ridge National Laboratory, Oak Ridge, TN 37831
6 Taras Shevchenko Kyiv National University, Radiophysical Faculty pr. Akademika Hlushkova 4g, 03022 Kyiv, Ukraine

Abstract

This work explores a nontrivial temperature behavior of the carriers concentration, which governs the conductance of the graphene channel on ferroelectric substrate with domain walls that is a basic element for field effect transistors of new generation. We revealed the transition from a single to double antiferroelectric-like hysteresis loop of the concentration voltage dependence that happens with the temperature increase and then exist in a wide temperature range (350 – 500) K. We have shown that the double loops of polarization and concentration can have irregular shape that remains irregular as long as the computation takes place, and the voltage position of the different features (jumps, secondary maxima, etc) changes from one period to another, leading to the impression of quasi-chaotic behavior. It appeared that these effects originate from the nonlinear screening of ferroelectric polarization by graphene carriers, as well as it is conditioned by the temperature evolution of the domain structure kinetics in ferroelectric substrate. The nonlinearity rules the voltage behavior of polarization screening by graphene 2D-layer and at the same time induces the motion of separated domain walls accompanied by the motion of p-n junction along the graphene channel. Since the domain walls structure, period and kinetics can be
controlled by changing the temperature, we concluded that the considered nano-structures based on graphene-on-ferroelectric are promising for the fabrication of new generation of modulators based on the graphene p-n junctions.

I. Introduction

The discovery of graphene by Novoselov and Geim [1, 2] had attracted great attention of the scientific community to the study of 2D structures, focused researchers on finding ways to minimize electronic devices and accelerated the transition to nanoelectronics [3, 4]. The unique properties of graphene make it possible to talk about its potential applications in nanoelectronics, optoelectronics, nanoplasmonics, and various sensoric devices, as evidenced by many publications, including the use of graphene for gas sensors [5], transparent electrodes for photovoltaic [6], the production of high-speed nonvolatile memory based on field-effect transistors (FET) with a graphene channel [7, 8, 9], and much more.

It is well known that the suspended graphene sheets, which have intriguing electronic properties [10, 11, 12, 13, 14, 15], cannot remain completely freestanding in space due to the twisting instability. Therefore, all studies associated with this 2D material are necessarily carried out in combination with supporting electrodes, holders or substrates of various kinds. Among the most common substrates are quartz (or mica) and ferroelectric substrates, at that a pronounced free charge accumulation can take place at the graphene - ferroelectric interface [16, 17]. A large number of studies had focused on layered heterostructures using graphene for non-volatile memory elements (see topical review [18] and refs. therein). However, they mainly addressed the macro characteristics (for the instrument as a whole) inherent in such systems, namely, the appearance of a hysteresis loop of the average concentration of carriers, the width of the memory window, the dependence of the memory window on the switching speed of the gate voltage. However, if we go down one level below and consider the behavior of the carriers when applying an external electric field, their distribution in graphene, the effect of the substrate on the distribution of carriers, then one can obtain more profound knowledge of the processes occurring in the system under applied external electric field. These studies are needed to understand and explain the macro-characteristics for such heterostructures.

One of these examples is the existence of a **p-n-junction** in a graphene channel, which was experimentally implemented on a dielectric SiO$_2$ substrate [19, 20], at first with the help of a multi-gate system [21]. P-N-junctions in graphene explore the opportunities to observe the physical manifestations of Andreev reflection, Klein tunneling, quantum Hall effect and Veselago lensing [22, 23, 24].
Later on, Hinnefeld et al. [25] and Baeumer et al. [26] created a p-n-junction in graphene using the ferroelectric substrates Pb(Zr, Ti)O$_3$ and LiNiO$_3$, respectively. In the case of a ferroelectric substrate, the formation of regions with electron and hole concentration of carriers in the channel arises due to the existence of domains with different directions of polarization [27, 28]. Kurchak et al. [29, 30] revealed that adsorbed charges dynamics leads to the hysteresis effect on conductivity in the graphene channel on an organic ferroelectric substrate. Kim et al. [31] propose graphene on ferroelectric for nonvolatile memory and reconfigurable logic-gate operations.

Morozovska et al. [32, 33, 34] have shown that the finite-size effects can strongly influence the nonlinear hysteretic dynamics of the stored charge and electro-resistance in the multilayer graphene on ferroelectric with domain stripes of different polarities, which can induce domains with $p$- and $n$-type conductivity, and with p-n-junction potentials at domain walls. Theoretical models for the different types of current regimes (from ballistic to diffusive one) in a single-layer graphene channel at 180°-ferroelectric domain walls have been developed [35, 36] and it was shown that the domain wall contact with the surface creates p-n junction in graphene channel. Recently thermodynamics and kinetics of the conductance of p-n-junctions induced in graphene channel by stripe domains nucleation, motion and reversal in a ferroelectric substrate has been explored using self-consistent approach based on Landau-Ginzburg-Devonshire (LGD) theory combined with classical electrostatics, semiconductor theory and quantum statistics for electrotransport calculations [37] at room temperatures.

Taking into account the above-mentioned theoretical results, it should be emphasized the importance and necessity of considering the dynamics of p-n junctions and the distribution of carrier concentrations along the graphene channel for various types of ferroelectrics in a wide temperature range. Such studies can answer the question about the possibility to create the devices based on graphene on ferroelectric heterostructures operating at different temperatures. To the best of our knowledge the dependence of graphene channel electro-conductance and resistance on ferroelectric substrate properties and domain structure evolution has been studied at room temperature only (see e.g. Refs.[16, 17, 25, 26, 31, 38, 39]). Specifically, hysteresis loops of graphene resistance, which have either ferroelectric-like or antiferroelectric-like shape depending on the top and bottom gate voltages, have been observed in the system graphene-on-ferroelectric substrate [16]. A qualitative and even semi-quantitative agreement of the loops calculated within self-consistent LGD approach with experiments [16, 38] was reached [33, 34]. However, the temperature evolution of the carrier concentration have not been studied experimentally or calculated theoretically.
Motivated by above problem, this work explores peculiarities of the carriers concentration temperature behavior, which governs the graphene channel conductance in the nano-structure graphene channel-ferroelectric substrate with domain structure that can be used as a basic element for FETs in advanced non-volatile memory units. We revealed the nontrivial dependences of the concentration hysteresis on temperature and gate voltage amplitude, which exist in the vicinity of ferroelectric transition temperature and far from it. It appeared that the nontrivial peculiarities originate from the temperature evolution of the domain structure kinetics in ferroelectric substrate.

II. Problem geometry and basic equations

System under consideration is similar to one examined in Ref.[37] and its geometry is shown in Fig.1(a). The geometry corresponds to the experiments [16, 17, 25, 26, 31, 38, 39] and includes top gate electrode, top oxide dielectric layer, conducting graphene channel with source and drain electrodes, ferroelectric substrate with top dead layer and bottom electrode.

![Figure 1](image.png)

Figure 1. (a) Schematics of the considered nano-hetero-structure consisting of top gate, dielectric oxide layer, graphene channel, ultra-thin paraelectric dead layer, ferroelectric substrate with domain walls and bottom gate electrode. (adapted from [37]) (b) Dependence of the 2D charge density of graphene on the dimensionless potential \( \psi \) calculated at different temperatures \( T_0 \) (black curve), \( T_1 = 1.5T_0 \) (red curve) and \( T_2 = 2T_0 \) (blue curve).

The technological requirements and realities of GFETs design are so that the existence of the top oxide and dead layers are unavoidable for most above mentioned experiments [16, 17, 25, 26, 31, 38, 39]. In particular the top oxide layer prevents graphene damage and contamination by the top gate electrode, as well as its thickness should be enough small and dielectric permittivity value should be enough high to prevent the dielectric breakdown [40, 41, 42]. The top oxide layer thickness vary in the range (5 – 50) nm, at that thinner layers should
have higher dielectric permittivity to minimize the probability of the breakdown. The relative dielectric permittivity of top oxide layer can vary in a wide range 4-100 depending on the oxide material [40-42], e.g. it is equal to 4 for SiO₂, 9 - 10 for Al₂O₃, 25-30 for HfO₂ and 80 for TiO₂.

The physical origin of natural dead layer is the unavoidable contamination of electrically-open (i.e. non-electroded) ferroelectric surface that becomes paraelectric aiming to provide maximally efficient electric screening of the polarization bound charge [43, 44]. There are different types of "dead" (or "passive") layers [43, 44], most of them are ultrathin sub-surface layers of thickness less than several nm located under the surface, where the spontaneous polarization is absent (or negligibly small) due to the surface reconstruction, contamination, zero extrapolation length and/or strong depolarization field [43, 44]. Artificial dead layers can be much thicker. The matter in the natural dead layer is of almost the same chemical composition than in the bulk of ferroelectric substrate, but it is in the paraelectric phase induced by the surface confinement effect, and so the relative dielectric permittivity of the dead layer ε₁ is usually rather high ~100 (as it should be for paraelectrics at room temperature [45]), in comparison with unity for a physical gap. Below the dead layer is treated as a top sub-surface layer of a ferroelectric film being in the paraelectric phase.

Above discussion means that the special attention should be paid to the choice of the dielectric properties and thicknesses of the top oxide and dead layers, because both of them strongly affect on electric potential and field distributions in graphene sheet and ferroelectric substrate. Additionally the dead layer controls the ferroelectric polarization screening and domain structure features [43, 44, 46, 47]. Anyway the electric potential drops in both these layers, and the drop depends on the layers relative dielectric permittivity ε₀ and ε₁, and thicknesses h₀ and h₁, respectively. In the oxide dielectric and ultrathin dead layers equations of state relating displacement D and electric field E, are \( D = \varepsilon_0 \varepsilon_o E \) and \( D = \varepsilon_0 \varepsilon_{DL} E \), \( \varepsilon_0 \) is a universal dielectric constant. Electric potentials φ₀ and φ₁ satisfies Laplace’s equation inside the layers. The dielectric permittivity of oxide layer ε₀ can vary in a wide range (4-100) depending on the oxide material. The relative dielectric permittivity ε₁ of the paraelectric dead layer is regarded rather high ~100. Appendix A presents the analytical (Eqs.(S.7)) and graphical (Figs.S1-S3) results of analytical study of the influence of the parameters h₀, H₁, ε₀ and ε₁ on the maximal spontaneous polarization value and graphene charge. To study the parameters h₀ and h₁, effect on the hysteresis loops we performed numerical calculations, which results are shown in Figs.S4. The parametric study allows us to select the values of parameters from their physical ranges listed in Table S1 for further numerical studies of the most interesting
temperature dependences of the carrier concentration and ferroelectric polarization in the considered nanostructure.

A planar single-layer graphene channel is characterized by the two-dimensional (2D) electron density of states, \( g_n(\varepsilon) = g_p(\varepsilon) = 2\varepsilon/(\pi\hbar^2 v_F^2) \) (see e.g. [48, 49]). Corresponding graphene charge density \( \sigma_G(\psi) = e(p_{2D}(\psi) - n_{2D}(\psi)) \) is the difference of 2D concentrations of electrons in the conduction band \( n_{2D}(\psi) = \int_0^\infty d\varepsilon g_n(\varepsilon) f(\varepsilon - E_F - e\varphi) \) and holes in the valence band \( p_{2D}(\varphi) = \int_0^\infty d\varepsilon g_p(\varepsilon) f(\varepsilon + E_F + e\varphi) \), respectively \( (E_F \) is a Fermi energy level, these expressions correspond the gapless graphene spectrum). Corresponding expression for \( \sigma_G(\psi) \) and its Pade-exponential approximation derived in Ref.[37] is

\[
\sigma_G(\psi) = \frac{4(k_B T)^2}{\pi\hbar^2 v_F^2} \sum_{m=-\infty}^{\infty} \frac{(-1)^m}{m^2} \sinh(m\psi) \approx \frac{2(k_B T)^2}{\pi\hbar^2 v_F^2} \left( \frac{1}{\eta(\psi)} - \frac{1}{\eta(-\psi)} \right),
\]

where the functions are \( \psi = \frac{e\varphi + E_F}{k_B T} \) and \( \eta(\psi) = \exp(\psi) + 2 \left( \psi^2 + \frac{\psi}{2} + \frac{2\pi^2}{12 - \pi^2} \right)^{-1} \). The dependence of \( \sigma_G(\psi) \) on the dimensionless voltage \( \psi \) is shown in Fig.1(b).

As a substrate for graphene channel we consider a ferroelectric film of thickness \( l \) with the spontaneous polarization component \( P^f_z \) directed along its polar axis \( z \). The film contains 180-degree domain wall – surface junctions [see Fig. 1(a)]. The dependence of the transverse polarization components \( P^f_i \) and \( P^f_2 \) on the field \( E \) are linear and corresponding relative dielectric permittivities are equal, \( \varepsilon_{11}^f = \varepsilon_{22}^f \). Polarization \( z \)-component is \( P^f_z(\mathbf{r}, E_3) = P^f_z(\mathbf{r}, E_3) + \varepsilon_0(\varepsilon_{33}^b - 1)E_3 \), where a so-called relative "background" permittivity \( \varepsilon_{33}^b \approx (4 - 7) \) is introduced [43, 50]. The distribution of \( P^f_z(x, y, z) \) is determined from the time-dependent LGD type Euler-Lagrange equation,

\[
\Gamma \frac{\partial P^f_3}{\partial t} + a(T)P^f_3 + b(P^f_3)^3 + c(P^f_3)^5 - g\Delta P^f_3 = E_3.
\]

\( \Gamma \) is a Landau-Khalatnikov relaxation coefficient [51], \( a(T) = \alpha_j(T - T_C) \), \( T \) is the absolute temperature, \( T_C \) is a Curie temperature of a bulk ferroelectric, \( b \) and \( c \) are the coefficients of LGD potential expansion on the polarization powers (also called as linear and nonlinear dielectric stiffness coefficients), \( g \) is a gradient coefficient and \( \Delta \) stands for a 3D-Laplace operator. The boundary conditions are of the third kind [52],
\[
\left(P_3^f - \Lambda_+ \frac{\partial P_3^f}{\partial z}\right)_{z=h_0} = 0, \quad \left(P_3^f + \Lambda_- \frac{\partial P_3^f}{\partial z}\right)_{z=h_{dl}+h_f} = 0 \quad (3)
\]

The physical range of extrapolation lengths \(\Lambda_{\pm}\) is (0.5 – 2) nm \(^{[53]}\). Quasi-static electric field is defined via electric potential as \(E_3 = -\partial \phi / \partial z\). The potential \(\phi_f\) satisfies Poisson equation inside a ferroelectric film.

For the problem geometry shown in the Fig. 1(a) the system of electrostatic equations acquires the form:

\[
\Delta \phi_O = 0, \quad \text{for} \quad -h_o < z < 0, \quad \text{(oxide dielectric layer "O") (4a)}
\]

\[
\Delta \phi_{DL} = 0, \quad \text{for} \quad 0 < z < h_{DL}, \quad \text{(dead layer "DL") (4b)}
\]

\[
\left(\varepsilon_{33}^b \frac{\partial^2}{\partial z^2} + \varepsilon_{11}^f \Lambda_+\right)\phi_f = \frac{1}{\varepsilon_0} \frac{\partial P_3^f}{\partial z}, \quad \text{for} \quad h_{DL} < z < h_{DL} + h_f. \quad \text{(ferroelectric "f") (4c)}
\]

3D-Laplace operator is \(\Delta\), 2D-Laplace operator on transverse coordinates \(\{x,y\}\) is \(\Delta_{\perp}\). Boundary conditions to Eqs.(4) are as follows: the fixed potential at the top gate, \(\phi_O(x,y,z = -h_o) = U(t)\) and zero potential at the bottom gate \(\phi_f(x,y,z = h_{DL} + h_f) = 0\); the continuity of the electric potential at the graphene layer, \(\phi_O(x,y,0) = \phi_{DL}(x,y,0)\); the equivalence of difference of the electric displacement normal components, and \(D_{3}^{DL} = \varepsilon_0 \varepsilon_{DL} E_3\), to the surface charges in graphene \(\sigma_G(x,y)\), \(D_{3}^O(x,y,0) - D_{3}^{DL}(x,y,0) = \sigma_G(x,y,\phi)\); and the continuity of the potential and displacement normal components, \(D_{3}^f = \varepsilon_0 \varepsilon_{33}^b E_3 + P_3^f\) and, at dead layer/ferroelectric interface, \(\phi_{DL}(x,y,h_{DL}) = \phi_f(x,y,h_{DL})\) and \(D_{3}^{DL}(x,y,h_{DL}) - D_{3}^f(x,y,h_{DL}) = 0\). Electric displacements are \(D_{3}^O = \varepsilon_0 \varepsilon_{O} E_3\), \(D_{3}^{DL} = \varepsilon_0 \varepsilon_{DL} E_3\) and \(D_{3}^f = \varepsilon_0 \varepsilon_{33}^b E_3 + P_3^f\). The gate voltage is periodic with a period \(T_g\), \(U(t) = U_{max} \sin(2\pi t/T_g)\). Periodic boundary conditions were applied in x-direction.

To resume this section we would like to underline that the problem (1)-(4) is coupled, because the LGD (2) and electrostatic equations (4) couple electric field and polarization. Actually the polarization \(P_3\) in a ferroelectric depends on the electric field \(E_3\) [see Eq.(2)]. In turn the field \(E_3\) strongly depends on the carrier concentration (1) in graphene \(\sigma_G(x,y,\phi)\) and dielectric layer properties via the boundary condition \(D_{3}^O(x,y,0) - D_{3}^{DL}(x,y,0) = \sigma_G(x,y,\phi)\). In accordance with Eq.(1) the concentration \(\sigma_G(x,y,\phi)\) in ruled by the electric potential \(\phi\), which in turn depends on the polarization value and its gradient according to Eqs.(4) with above-listed electric boundary conditions. Hence the coupled equations (1)-(4) self-consistently describe the influence of electric field induced by the ferroelectric domain structure on the graphene.
conductivity, as well as the influence of graphene top electrode on the domain structure in a ferroelectric. The direct sequences of the mutual graphene-ferroelectric influence are (i) the appearance of p-n junctions in graphene at FDW [30, 35, 36, 37], (ii) the broadening of the domain walls near the ferroelectric surfaces [54] appearing to minimize the depolarization electric field energy [as shown in Fig.1(a)] and unusual temperature evolution of the ferroelectric polarization switching and hysteresis loops of charge density in graphene, which will be presented and analyzed in next sections.

Notably the used self-consistent approach "LGD + electrostatics" is well-recognized for calculations of local polarization dynamics at ferroelectric interfaces [55]. The approach coupling ferroelectric polarization, electric field and surface and/or space charge are the basis for phase-field simulations and finite element modelling of the polarization reversal and domain structure evolution in ferroelectric nano-grained ceramics, thin films and their graded structures (see e.g. seminal papers of L.Q. Chen group [56, 57], P. Alpay group [58, 59, 60] and J. F. Scott group [61, 62]).

III. Influence of ferroelectric domain structure on the carrier concentration in graphene channel at different temperatures

Below we present results of numerical modeling of the problem (1)-(4). We study numerically the modulation of the carrier concentration, which governs the graphene channel conductance caused by a domain structure moving in a ferroelectric substrate. Parameters used in the calculations are listed in Table SI.

Plots in Fig. 2 show the spatial distribution of polarization in a 75-nm thick ferroelectric substrate calculated at several temperatures within the range (300 – 500)K and gate voltages \( U_{\text{max}} = 2 \text{V} \) (a) and 10 \( \text{V} \) (b). From Figs.2 the maximal polarization and domain structure contrast decreases with temperature increase, and the broadening of the domain walls increases near the substrate-gap interface. From Fig.2(a) the domain period decreases with temperature increase. At 300 K the domain structure is relatively contrast in comparison with the faint image at 500 K for \( U_{\text{max}} = 2 \text{V} \). The domain period is almost temperature-independent in Fig.2(b), but ferroelectric domain structure disappears at 500 K for \( U_{\text{max}} = 10 \text{V} \).
Figure 2. Spatial distribution of a ferroelectric polarization component $P_3$ calculated for different temperatures 300, 350, 400, 450 and 500 K (legends at the plots); Snapshot of domain structure was done at moment of time $t=T_g=1000$ s. The gate voltage amplitude $U_{\text{max}} = 2$ V (a) and $U_{\text{max}} = 10$ V (b). Parameters are listed in Table SI.

The nontrivial question to Fig. 2 is why ferroelectric domain structure disappears at 500 K for $U_{\text{max}} = 10$ V, but still exists at $U_{\text{max}} = 2$ V? A possible answer is that applied voltage increases the degree of the substrate unipolarity and supports the ferroelectric single-domain state [37]. Note that the transition temperature of the single-domain ferroelectric substrate into a non-polar paraelectric phase decreases under the thickness decrease; it is about 500 K for a 70 nm film at $U_{\text{max}} = 0$ and chosen material parameters. The critical thickness is much smaller for the poly-domain films [63]. So that increasing $U_{\text{max}}$ leads to the film unipolarity and increases its critical thickness at the same time.

The spatial-temporal evolution of the domain structure defines the evolution of the carrier concentration $\Delta n_c(x,t)$ in the graphene channel; at that a separate p-n junction is located above
each domain wall [35-37]. To illustrate this, Figs 3 show x-profiles of the $\Delta n_G(x,t)$ calculated along the graphene channel at several temperatures within the range (300 – 500)K and gate voltages $U_{\text{max}} = 2V$ (a) and $10V$ (b). Positive $\Delta n_G(x,t)$ correspond to holes, and negative to electrons. The amplitude of $\Delta n_G(x,t)$ is maximal for room temperature 300 K, monotonically and strongly decreases with temperature increasing; then it almost [Fig.3(a)] or completely [Fig.3(b)] vanishes at 500 K. The amplitude decrease is in agreement with the temperature decrease of maximal polarization in the central regions of domains, $P_s \sim \sqrt{T_C - T}$. The modulation amplitude is almost the same at $U_{\text{max}} = 2V$ and $10V$.

There are two p-n junctions at (300-350) K located at distance 15 nm; and four p-n junctions at (400-450) K at $U_{\text{max}} = 2V$ [Fig.3(a)]. The temperature-dependent number of p-n junctions is determined by the corresponding number of domain walls, which doubles with the temperature increase from 300 K to 400 K [see Fig.3(a)]. For the reasons described above [see Fig.2(b) and comments] the amount of p-n junctions does not change with temperature increase from 300 to 450 K at $U_{\text{max}} = 10V$ [see black, red, magenta and blue curves in Fig.3(b)]. The charge modulation disappears at 500 K, since the domain structure disappears [see the green line in Fig.3(b) and plot for 500 K in Fig.2(b)].

**Figure 3.** 2D-concentration of the carriers $\Delta n_G(x,t)$ in the graphene channel calculated for different temperatures 300, 350, 400, 450 and 500 K (different curves) at time moment $t=T_g=1000$ s. The gate voltage amplitude $U_{\text{max}} = 2V$ (a) and $U_{\text{max}} = 10V$ (b). P-N junctions are marked by circles. Parameters are listed in Table SI.
Color maps in Figs 4(a) and 4(b) illustrate the temporal evolution of the 2D-concentration of carriers x-profile in graphene channel \( (\Delta n_G(x,t)) \) for the temperatures 300 K and 400 K, respectively. It is seen that the periodic changes of \( \Delta n_G(x,t) \) correlate with the number of domain walls in a ferroelectric substrate that doubles with temperature increasing from 300 K to 400 K. At that the amplitude of \( \Delta n_G(x,t) \) decreases in an order of magnitude (from \( 10^{18} \text{m}^{-2} \) to \( 10^{17} \text{m}^{-2} \)) with the temperature increasing from 300 K to 400 K.

![Figure 4](image-url)

**Figure 4.** Spatial-temporal evolution of the 2D-concentration of carriers in graphene channel calculated for different temperatures 300 K (a) and 400 K (b). The gate voltage amplitude \( U_{\text{max}} = 2 \text{ V} \). Color scale changes from \( \pm 10^{18} \text{m}^{-2} \) for 300 K to \( \pm 10^{17} \text{m}^{-2} \) for 400 K. Parameters are listed in Table SI.

Color maps in Figs 5(a) and 5(b) show the temperature changes of the 2D-carriers x-profile calculated for two temperature ranges, (300 – 400) K and (300 – 560) K, respectively. It is seen that the contrast of \( \Delta n_G(x,t) \) monotonically decreases in an order of magnitude (from \( 10^{18} \text{m}^{-2} \) to \( 10^{17} \text{m}^{-2} \)) with the temperature increasing from 300 K to 560 K. The number of minima and maxima correlates with the number of domain walls in a ferroelectric substrate that doubles with temperature increasing from 300 K to 400 K [see Fig. 3(a) and 3(b)].
Figure 5. Spatial-temperature distribution of the 2D-concentration of carriers in graphene channel calculated for the temperature ranges (300 − 400) K (a) and (400 − 560) K (b). The gate voltage amplitude $U_{\text{max}} = 2$ V. Color scale changes from $\pm 10^{18}$ m$^{-2}$ for 300 K to $\pm 10^{17}$ m$^{-2}$ for 400 K. Parameters are listed in Table SI.

IV. Hysteretic response of average carrier concentration in graphene channel

The dependence of the average concentration of carriers in graphene channel on the voltage applied to the gate is hysteretic and temperature-dependent [see the right column in Fig.6]. At that the concentration loops are very different from the average polarization loops at small amplitudes of the gate voltage $U_{\text{max}} \leq 10$ V [compare Fig.6(a) with 6(b)]. In particular the concentration loop has a pronounced single hysteresis shape with ending characteristic for the conductivity contribution at room temperature and $U_{\text{max}} = 10$ V. The loop becomes lower and slightly thinner at 450 K, then it transforms into a very slim double-shape loop at 500 K (see different loops in Fig.6(b) and Ref.[37]). Corresponding polarization loops are very slim in the actual temperature and gate voltage range $U_{\text{max}} \leq 10$ V as anticipated for a multi-domain scenario of polarization reversal (see different loops in Fig.6(a) and Ref.[37]). Hence the transformation of the concentration loop with the temperature increase is dictated by the domain structure dynamics at $U_{\text{max}} \leq 10$ V, rather then by the ferroelectric polarization loop changes, and the relation occurs in a non-proportional way.

Notably the polarization loop shape changes from the slim one and to a pronounced square-like hysteresis with the gate voltage amplitude increase to 15V and temperatures $T \leq 320$ K [see orange and red loops in Figs. 6(c)]. At $T \leq 320$ K the temperature behavior of
square-like concentration loops is dictated by the almost single-domain polarization reversal at $U_{\text{max}} \geq 10$ V, but the relation occurs in a non-proportional way [see orange and red loops in Figs. 6(d)].

With the temperature increase above 320 K the double antiferroelectric-like hysteresis loops of polarization and concentration appear, undergoing unusual shape changes (additional flexures and maxima) and exist in a wide temperature range $320 \leq T \leq 500$ K [compare magenta, blue, dark green and light green loops in Figs. 6(c) and 6(d)]. The height and width of the double loops decreases with the temperature increase, and minor loops by-pass is counterclockwise [see magenta, blue, dark green and light green loops in Figs. 6(e) and 6(f)]. These double antiferroelectric-like loops of polarization and concentration disappear at approximately (500 - 550) K indicating the transition of the ferroelectric substrate to a paraelectric phase [see green and black linear curves in Figs. 6(c)-(f)].

Note that the wide temperature range of the double loop existence (about 180 K) is at least 10 times more that the range of double hysteresis loops existence in the ferroelectric with the first order phase transition to a paraelectric phase. The physical origin of the double antiferroelectric-like loops can be explained by the voltage behavior of the graphene charge density shown in Fig. 1(b) for different temperatures. The dependence becomes significantly steeper with the temperature decrease. Since the graphene charge is responsible for the polarization screening in a ferroelectric substrate, high charge densities (corresponding to high acting electric potential $|\varphi| >> k_B T$ and lower temperatures) can provide an effective screening, and the low ones (corresponding to small potentials $|\varphi| \leq k_B T$ and higher temperatures) can provide a weak incomplete screening only. At low enough temperatures ($T \leq 320$ K) the substrate (PZT film of thickness 75 nm) is in a deep ferroelectric phase, and relatively weak screening of its depolarization field by graphene sheet is enough to support the ferroelectricity in it. Corresponding hysteresis loop is a single one, has a square-like shape with relatively high remanent polarization and coercive voltage. The situation principally changes with the temperature increase, since the film approaches the paraelectric transition. The transition happens at 666 K for a bulk sample, for thin films it happens at much lower temperatures and strongly depends on the film thickness and screening conditions in a self-consistent way [63, 64, 65, 66]. At that the better the screening the higher is the critical thickness of the transition [63, 65, 66]. Thus additional screening by graphene carriers is urgently required to maintain the thin film in a ferroelectric state. As one can see from Fig. 1(b) the screening increase appears at nonzero potential $\varphi$ that is in turn proportional to the gate voltage $U$. The critical voltage corresponding to the screening degree enough to suppress the thickness-induced paraelectric
transition opens the minor loops of polarization, which in turn induce the concentration loops of anti-hysteretic type.

**Figure 6.** Hysteresis loops of the average polarization $P(U)$ of ferroelectric substrate (a, c, e) and carriers concentration variation in graphene channel $\Delta n_G(U)$ (b, d, f) calculated for different temperatures (different curves), $U_{\text{max}} = 10$ V (a, b) and $U_{\text{max}} = 15$ V (c, d, e, f); $T_g=1000$ s. Other parameters are listed in Table SI. Arrows indicate the counterclockwise direction of the loops by-pass.
Plots (e) and (f) show positive one-half of the double hysteresis loops calculated at 350, 400, 450, 500 and 550 K (different curves), $U_{\text{max}} = 15$ V.

Single loops of polarization and conductivity (such as shown in Figs.6), which correspond either to poly-domain [Fig.6(a,b)] or to the monodomain [Fig.6(c,d)] scenario of polarization reversal, always have regular shape [see Figs.7(a)]. However the double loops, which have regions with poly-domain states at definite temperatures, can be or regular or irregular shape [compare Figs.7(b) and 7(c)].

Single and double hysteresis loops, which have a regular shape (symmetric or anti-symmetric), are reproducible at every period of applied voltage, as anticipated. Quite unexpectedly we revealed the double hysteresis loops of polarization and concentration, which have irregular shape, are "superperiodic" in the sense that they are reproduced only after a great amount of the voltage periods [Figs.7(c)]. An example of ten period "superperiodic" polarization and concentration double loops of irregular shape are shown in Figs.7(d) and Figs.7(e), respectively. The wording "superperiodicity" in the considered case means that the double loop shape remains irregular as long as the computation takes place, and the voltage position of the different features (jumps, secondary maxima, etc.) changes from one period to another, leading to the impression of quasi-chaotic behavior. We established that the physical origin of the "quasi-chaotic" behavior of the polarization and concentration double loops is the strongly nonlinear voltage behavior of the graphene screening charge [see Eq.(1) and Fig.1(b)]. The nonlinearity rules the voltage behavior of polarization screening by graphene 2D-layer and at the same time induces the motion of separated domain walls accompanied by the motion of p-n junction along the channel. However these speculations do not account for the lattice pinning effect on the domain wall motion, that can decrease the velocity of domain walls on several orders of magnitude [67]. On the other hand the local excitation by the probe electric field opens the possibilities for intrinsic switching of polarization [68, 69] and correlated nucleation of domains [70] and so the quasi-free motion of domain walls are far not excluded in thin films.

To quantify the anomalous temperature behavior of the polarization and graphene carriers concentration loops we calculated the temperature dependences of the switchable remanent polarization $2P_r$ (polarization loop height at moments of time which correspond to zero gate voltage value) and concentration $2\Delta n_r$ (concentration loop height at moments of time which correspond to zero gate voltage value) and coercive voltage windows, $2U_{CP}$ and $2U_{CN}$, respectively corresponding to the width of the loops [see designations in Figs.7(a)]. For double loops we calculated the temperature dependence of the critical voltage of the minor loops opening, $U_{cr}$ [see designations in Figs.7(b)].
Calculation results for polarization and concentration loops parameters are shown in Figs.8 and 9, respectively. As one can see from the figures, there are functional proportionally between the parameters temperature behavior, namely $P_r(T) \sim \Delta n_r(T)$ and $U_{CR}(T) \sim U_{CN}(T)$, moreover $U_{CR}(T) = U_{CN}(T)$ for a single loops. However for double loops $U_{CR}(T) < U_{CN}(T)$ on a value about 5-2 V, and the difference is temperature dependent [see Fig.10].

Different curves in Figs. 8-9 correspond to different amplitudes of the gate voltage $U_{max} = (2, 5, 8, 10, 15)$ V. Also one can see the pronounced peculiarity related with the transition from the single-domain polarization reversal to a poly-domain one which occurs at $U_{max} = 15$ V and $T \approx 330$ K (see green curves in Figs.8-9). The abrupt at black, red, blue and magenta curves corresponds to the transition from the single loop to the double one. The steps at coercive voltages correspond to the domain structure re-building (such as e.g. abrupt changes of its period) taking place due to the spatial confinement of the film in transverse direction. The zigzag-like oscillations of on the polarization, concentration and their coercive fields temperature dependences originated from the quasi-chaotic behavior of the double loops with irregular shape [such as shown in Figs.7(d)-(e)]. Namely, the oscillations correspond to the left- and right-voltage asymmetry of the double loops shape.

Figure 7. Schematics showing the parameters of polarization $P(U)$ (a) and concentration $\Delta n(U)$ (b) hysteresis loops. Graph (c) schematically shows double loops of irregular shape. Multistable quasi-
chaotic double loops of polarization (d) and concentration (e) calculated at $T=420\text{K}$ and $U_{\text{max}} = 15\text{ V}$. The loops "super-period" consisting of 10 period of the gate voltage.

**Figure 8.** Temperature dependences of the remanent average polarization $P_R(U)$ (a, b), and coercive voltage of polarization reversal (c, d) calculated for the different amplitudes of the gate voltage $U_{\text{max}} = (2, 5, 8, 10, 15)\text{ V}$ and $T_g=1000\text{ s}$. Other parameters are listed in Table SI. Abbreviation "DSR" indicates the region of domain structure rebuilding. Regularly changing parameters (regular curves) of the single loops and oscillating parameters (randomly oscillating curves) of the double loops with regular or irregular shape are shown.
Figure 9. Temperature dependences of the remanent concentration of the carriers in graphene channel $\Delta n_R (U)$ (a, b) and coercive voltage of concentration reversal (c, d) calculated for the different amplitudes of the gate voltage $U_{max} = (2, 5, 8, 10, 15)$ V and $T_g = 1000$ s. Other parameters are listed in Table SI. Abbreviation "DSR" indicates the region of domain structure rebuilding. Regularly changing parameters (regular curves) of the single loops and oscillating parameters (randomly oscillating curves) of the double loops are shown.
Figure 10. Temperature dependences of the coercive voltage of polarization (curves with labels $U_{CP}^{\min}$ and $U_{CP}^{\max}$) and concentration (curves with labels $U_{CN}^{\min}$ and $U_{CN}^{\max}$) reversal calculated for the maximal amplitudes of the gate voltage $U_{max} = 15$ V and $T_g = 1000$ s. Other parameters are listed in Table S1. The zigzag-like oscillations of on the coercive fields temperature dependences originated from the quasi-chaotic behavior of the double loops with irregular shape.

Note, that the study of the temperature dependence of graphene channel conductance involves the problem how the carriers mean free path depends on temperature [3]. However, for the most common case of dominant carriers scattering mechanism in the channel at the ionized impurities in the substrate it can be demonstrated (see e.g. [3, 28]) that the channel conductance follows the general features of temperature dependence, presented by hysteresis loops in Figs. 6-7, and their parameters, analyzed in Figs. 8-10. This temperature behavior of the graphene channel conductivity should be taken into account in GFETs operation.

Conclusions

We studied how the temperature evolution of the domain structure in a ferroelectric substrate influences on the distribution of the concentration and type of carrier’s in a graphene channel. It was found that the average concentration of carriers in graphene channel reveals the behavior similar to ferroelectric (single hysteresis loops) or antiferroelectric-like (double hysteresis loops) depending on the gate voltage amplitude and temperature. At that the wide temperature range of the double loop existence (about 180 K) is at least 10 times more that the range of double
hysteresis loops existence in the ferroelectric with the first order phase transition to a paraelectric phase.

The physical origin of the single to double antiferroelectric-like loop transition can be explained the voltage behavior of the graphene charge density shown for different temperatures, at that density becomes significantly steeper with the temperature decrease [Fig.1(b)]. The graphene charge is responsible for the polarization screening in a ferroelectric substrate. High charge densities (corresponding to high acting electric potential and lower temperatures) can provide an effective screening, and the low ones (corresponding to small potentials and higher temperatures) can provide a weak incomplete screening only. At room and lower temperatures the substrate (PZT film of thickness 75 nm) is in a deep ferroelectric phase, and relatively weak screening of its depolarization field by graphene sheet is enough to support the ferroelectricity in it. Corresponding hysteresis loop is a single one, has a square-like shape with relatively high remnant polarization and coercive voltage. The situation principally changes with the temperature increase, since the film approaches the paraelectric transition. At that the better the screening the higher is the critical thickness of the transition [64-66]. Thus additional screening by graphene carriers is urgently required to maintain the thin film in a ferroelectric state. As one can see from Fig.1(b) the screening increase emerges at nonzero potential $\phi$ that is in turn proportional to the gate voltage $U$. When the critical voltage corresponds to the screening degree enough to suppress the thickness-induced paraelectric transition, it opens minor loops of polarization, which in turn induce the concentration loops of anti-hysteretic type.

Single and double loops, which have a regular shape, are reproducible at every period of applied voltage, as anticipated. Unexpectedly we revealed the double loops of polarization and concentration, which have irregular shape. They remains irregular as long as the computation takes place, and the voltage position of the different features (jumps, secondary maxima, etc.) changes from one period to another, leading to the impression of quasi-chaotic behavior [Figs.7(d,e)]. We established that the physical origin of the "quasi-chaotic" behavior of the polarization and concentration double loops is the strongly nonlinear voltage behavior of the graphene screening charge. The nonlinearity rules the voltage behavior of polarization screening by graphene 2D-layer and at the same time induces the motion of separated domain walls accompanied by the motion of p-n junction along the graphene channel. More detailed analysis of the nonlinear system behavior should be performed within the framework of bifurcation theory [71] that will be the subject of our further studies.

Existence of multi-domain states in a ferroelectric substrate causes inhomogeneous carrier’s distribution in graphene channel and lead to formation a set of p-n junctions along
channel. Revealed p-n junctions exist in a wide temperature range below the ferroelectric transition temperature; they vanish in the immediate vicinity of the temperature.

Since the domain walls structure, period and kinetics can be controlled by varying the temperature, we concluded that the considered nano-structures based on graphene-on-ferroelectric elements are promising for the fabrication of new generation of modulators and non-volatile memory units based on the graphene p-n junctions.

Acknowledgements. The publication contains the results of studies conducted by President’s of Ukraine grant for competitive projects (grant number F74/25879) of the State Fund for Fundamental Research (A.I.K., A.N.M.). A.N.M. acknowledges the European Union’s Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No 778070 and the National Academy of Sciences of Ukraine (projects No. 0118U003535 and 0117U000240). S.V.K. acknowledges the Office of Basic Energy Sciences, U.S. Department of Energy. Part of work was performed at the Center for Nanophase Materials Sciences, which is a DOE Office of Science User Facility.

Authors' contribution. A.I.K. and E.A.E. wrote the codes, performed numerical calculations and presented their results in graphical form, as well as assisted A.N.M. with the interpretation of numerical results. A.N.M. generated the research idea, stated the problem and wrote the manuscript. S.V.K. and M.V.S. working on the manuscript improvement and discussion.
Appendix A. Parametric study of the electric fields

The problem geometry shown in the Fig. 1(a) the system of electrostatic equations acquires the form:

\[ \Delta \varphi_O = 0, \quad \text{for} \quad -h_O < z < 0, \quad \text{(oxide dielectric layer "O")} \quad (S.1a) \]

\[ \Delta \varphi_{DL} = 0, \quad \text{for} \quad 0 < z < h_{DL}, \quad \text{(dead layer "d")} \quad (S.1b) \]

\[ \left( \frac{\varepsilon_{33}}{\varepsilon_i} \frac{\partial^2}{\partial z^2} + \varepsilon_{11} \Delta \right) \varphi_f = \frac{1}{\varepsilon_o} \frac{\partial P_f^0}{\partial z}, \quad \text{for} \quad h_{DL} < z < h_{DL} + h_F. \quad \text{(ferroelectric "f")} \quad (S.1c) \]

3D-Laplace operator is \( \Delta \), 2D-Laplace operator is \( \Delta_z \). Explicit form of the boundary conditions (BCs) in z–direction is:

\[ \varphi_O (x,y,-h_O) = U(t), \quad (S.2a) \]

\[ \varphi_O (x,y,0) = \varphi_{DL} (x,y,0), \quad D^f_j (x,y,0) - D^f_j (x,y,0) = \sigma_G (x,y,\varphi), \quad (S.2b) \]

\[ \varphi_{DL} (x,y,h_{DL}) = \varphi_f (x,y,h_{DL}), \quad D^f_j (x,y,h_{DL}) - D^f_j (x,y,h_{DL}) = 0, \quad (S.2c) \]

\[ \varphi_f (x,y,h_{DL} + h_F) = 0. \quad (S.2d) \]

Let us consider 1D case and suppose that distribution is almost homogeneous inside a wide domain, \( P_3 \approx \left\langle P_3 \right\rangle \), where the averaging here means the averaging over the domain volume. The assumption is also valid for e.g. single-domain ferroelectric state, or in the central region of a wide domain the high extrapolation length \( \Lambda_z \gg \sqrt{\varepsilon_0 \varepsilon_{12} G} \). For the case the solution for potential distribution is:

\[ \varphi_O = U - E_O (z - h_{DL} - h_O), \quad \varphi_{DL} = \Psi - z E_{DL}, \quad \varphi_f = -(h_F + z) E_f. \quad (S.3) \]

Here we introduced designations \( E_O, E_{DL} \) and \( E_f \) for an electric fields in the top oxide layer, lower dead layer and ferroelectric film. Using the conditions of potentials continuity at the interfaces, one could express the electric field via interface potentials \( \Phi \) and \( \Psi \) as follows:

\[ E_O = \frac{\Phi - U}{h_O}, \quad E_{DL} = \frac{\Psi - \Phi}{h_{DL}}, \quad E_f = \frac{-\Psi}{h_f}. \quad (S.4) \]

The interface potentials \( \Phi \) and \( \Psi \) should be determined from the conditions of displacement continuity:

\[ \varepsilon_0 \varepsilon_{DL} \frac{\Psi - \Phi}{h_{DL}} = \left\langle P_3 \right\rangle - \varepsilon_0 \varepsilon_{b} \frac{\Psi}{h_f}, \quad \varepsilon_0 \varepsilon_O \frac{\Phi - U}{h_O} - \varepsilon_0 \varepsilon_{DL} \frac{\Psi - \Phi}{h_{DL}} = \sigma(\Phi). \quad (S.5) \]
Averaging of the LGD equation (2) and application of the boundary conditions (3) gives

\[
h_f \left\{ a_5 P_3 + a_{33} P_3^3 + a_{333} P_3^5 \right\} + g \left( \frac{P_3}{\Lambda_{zz=0}} + \frac{P_3}{\Lambda_{zz=h_f}} \right) = h_f E_f \quad \text{and at} \quad \Lambda_{zz} \gg \sqrt{\varepsilon_0 e_{333} g}
\]

it yields to

\[
a_5 \langle P_3 \rangle + a_{33} \langle P_3 \rangle^3 + a_{333} \langle P_3 \rangle^5 \approx -\frac{\Psi}{h_f}.
\]

Then the system of nonlinear Eqs.(S.4)-(S.6) and Eq.(1) for \( \langle P_3 \rangle \), \( \Phi \) and \( \Psi \):

\[
\Phi = \frac{h_{DL}}{\varepsilon_{DL}} \left( -\frac{\langle P_3 \rangle}{\varepsilon_0} + \frac{\varepsilon_h}{h_f} + \frac{\varepsilon_{DL}}{h_f} \right) \Psi,
\]

\[
U = \left( 1 + \frac{\varepsilon_h}{h_f} \left( \frac{h_{DL}}{\varepsilon_{DL}} + \frac{h_0}{\varepsilon_0} \right) \right) \Psi - \left( \frac{h_{DL}}{\varepsilon_{DL}} + \frac{h_0}{\varepsilon_0} \right) \frac{\langle P_3 \rangle}{\varepsilon_0} - \frac{h_0 \sigma_G(\Phi)}{\varepsilon_0},
\]

\[
\sigma_G(\Phi) = \frac{4(k_B T)^2 e}{\pi h^2 v_F} \sum_{m=1}^{\infty} \frac{(-1)^m}{m^2} \sinh \left[ m \left( \frac{e\Phi}{k_B T} \right) \right] \approx \frac{2(k_B T)^2 e}{\pi h^2 v_F} \left( \frac{1}{\eta(\Phi)} - \frac{1}{\eta(-\Phi)} \right),
\]

\[
a_5 \langle P_3 \rangle + a_{33} \langle P_3 \rangle^3 + a_{333} \langle P_3 \rangle^5 \approx -\frac{\Psi}{h_f}.
\]

The asymptotic series for \( \sigma_G(\Phi) \) is converging until \( \left| \frac{e\Phi}{k_B T} \right| < 1 \), while its approximation with

\[
\eta(\psi) = \exp(\psi) + 2 \left( \psi^2 + \frac{\psi}{2} + \frac{2\pi^2}{12 - \pi^2} \right)^{-1}
\]

is valid for all \( \Phi \).

From Eqs.(S.7) spontaneous polarization \( P_3(U = 0) \) induces the charge density \( \sigma_G(\Phi) \) in graphene. Equations (S.7) allow us to control the value of electric fields in the oxide and dead layers, since they should be much smaller than the threshold breakdown fields, which are less than 1 V/nm for oxide dielectric (e.g. SiO\textsubscript{2}) and less than 0.1 V/nm for a paraelectric dead layer.

Below we performed numerical calculations for the parameters listed in Table S1.

**Table S1.** Material parameters used in numerical calculations

<table>
<thead>
<tr>
<th>Parameter, constant or value</th>
<th>Numerical value and dimensionality</th>
</tr>
</thead>
<tbody>
<tr>
<td>oxide dielectric thickness</td>
<td>( h_o = 8 ) nm (typical range 5 – 50 nm)</td>
</tr>
<tr>
<td>natural dead layer thickness</td>
<td>( h_{DL} = 0.4 ) nm (typical range 0.2 – 2 nm)</td>
</tr>
<tr>
<td>ferroelectric film thickness</td>
<td>( h_{F} = 75 ) nm (typical range 10 nm – 10 ( \mu )m)</td>
</tr>
<tr>
<td>graphene channel length</td>
<td>( L = 200 ) nm (32 nm region is shown in figures)</td>
</tr>
</tbody>
</table>
universal dielectric constant \[ \varepsilon_0 = 8.85 \times 10^{-12} \text{ F/m (e/Vm)} \]

permittivity of the oxide dielectric layer \[ \varepsilon_O = 20 \text{ (typical range 10 – 300)} \]

permittivity of the dielectric dead layer \[ \varepsilon_{DL} = 100 \text{ (typical range 50 – 500)} \]

Ferroelectric permittivity of the ferroelectric film \[ \varepsilon_{f}^3 = 500, \varepsilon_{11}^f = \varepsilon_{22}^f = 780 \text{ (Pb(ZrTi)O}_3\text{-like)} \]

Background permittivity of the ferroelectric film \[ \varepsilon_{11} = \varepsilon_{22} = \varepsilon_{33} = 4 \text{ (Pb(ZrTi)O}_3\text{, BaTiO}_3\text{, or other ferroelectric perovskites)} \]

Landau-Ginzburg-Devonshire potential coefficients \[ \alpha_T = 2.66 \times 10^8 \text{ C}^2 \cdot \text{mJ/K, } T_C = 666 \text{ K (PbZr}_x\text{Ti}_1-x\text{O}_3, x \approx 0.5), P_s = (0.5 - 0.7) \text{ C/m}^2, c = 8.02 \times 10^8 \text{ J C}^{-6} \cdot \text{m}^6 \]

Temperature \[ T = 300 \text{ K; 350 K; 400 K; 450 K; 500 K} \]

dielectric anisotropy of ferroelectric film \[ \gamma = \sqrt{\varepsilon_{33}/\varepsilon_{11}} = 0.8 \]

extrapolation length \[ \Lambda_+ = \Lambda_- = \infty \]

Plank constant \[ \hbar = 1.056 \times 10^{-34} \text{ J-s} = 6.583 \times 10^{-16} \text{ eV-s} \]

Fermi velocity of electrons in graphene \[ v_F \approx 10^6 \text{ m/s} \]

The "reduced" thickness of oxide layer \( h_O/\varepsilon_O \) is small and the "reduced" thickness of dead layer \( h_{DL}/\varepsilon_{DL} \) is very small for physically reasonable thicknesses \( h_{O,DL} \) and relative permittivities \( \varepsilon_{O,DL} \) ranges listed in Table S1. Figures S1-S3 illustrate the dependences of the positive solution for spontaneous polarization \( \langle P_3 \rangle \) and induced negative charge \( \sigma_G \) on the parameter \( h_O/\varepsilon_O \), calculated for several values of the ratio \( h_{DL}/\varepsilon_{DL} \) varying in the range (0.002 – 0.03), temperatures \( T=300, 350, 400 \text{ and 450 K} \). Negative solutions for spontaneous polarization \( -\langle P_3 \rangle \) and positive graphene charge \( -\sigma_G \) are not shown, since the curves are symmetric to the curves \( +\langle P_3 \rangle \) and \( +\sigma_G \) with respect to x-axes.

One can see from the Figs.S1 and S3(a) that \( \langle P_3 \rangle \) decreases with \( h_O/\varepsilon_O \) increasing. However the dependence of \( \langle P_3 \rangle \) on \( h_O/\varepsilon_O \) becomes non-monotonic and then pronouncedly bistable (two positive values of \( \langle P_3 \rangle \) corresponds to one value of \( h_O/\varepsilon_O \)) with \( h_{DL}/\varepsilon_{DL} \) decrease and temperature decrease [compare Figs.S1(a)-(d)]. The polarization disappears when the ratio \( h_O/\varepsilon_O \) becomes higher that some critical value, which in turn increases with the ratio \( h_{DL}/\varepsilon_{DL} \) decrease.

One can see from the Figs.S2 and S3(b) that \( \sigma_G \) firstly increases and then decreases with \( h_O/\varepsilon_O \) increasing. Hence the dependence of \( \sigma_G \) on \( h_O/\varepsilon_O \) is always non-monotonic and becomes pronouncedly bistable with temperature decrease (two negative values of \( \sigma_G \) corresponds to one value of \( h_O/\varepsilon_O \)) with \( h_{DL}/\varepsilon_{DL} \) decrease [compare Figs.S2(a)-(d)]. The
graphene charge disappears when the ratio $h_o/\varepsilon_o$ becomes higher that some critical value, which in turn increases with the ratio $h_{DL}/\varepsilon_{DL}$ decrease.

Let us underline that the carrier density in graphene induced by ferroelectric substrate can be substantially lower than the polarization of the ferroelectric layer (compare $\langle P_3 \rangle = 0.5 \text{ C/m}^2$ and $\sigma_g = -0.3 \text{ C/m}^2$ at 300 K; $\langle P_3 \rangle = 0.4 \text{ C/m}^2$ and $\sigma_g = -0.03 \text{ C/m}^2$ at 500 K).

Notably the bistability regions of $\langle P_3 \rangle$ and $\sigma_g$ corresponds to the appearance of contraction and other complex features at the polarization and charge hysteresis loops, at that the charge bistability regions are much wider and pronounced. The bistability regions are wider and more pronounced at room and lower temperatures; they becomes narrow and vanishes with the temperature increase (compare plots (a) for 300 K with plots (b)-(d) for higher temperatures).

From Figs. S1(a) and S2(a) the bistability region of spontaneous polarization and graphene charge appears for the ratio $h_o/\varepsilon_o$ within the range $0.03 \text{nm} < h_o/\varepsilon_o < 1 \text{nm}$ at 300 K. The bistability region exists at $h_{DL}/\varepsilon_{DL} < 0.03 \text{ nm}$.

Hence we select the values $h_{DL} = 0.4 \text{nm}$ and $\varepsilon_{DL} = 100$ (corresponding to $h_{DL}/\varepsilon_{DL} = 0.004 \text{ nm}$), $h_o = 0.8 \text{ nm}$ and $\varepsilon_o = 20$ (corresponding to the ratio $h_o/\varepsilon_o = 0.04 \text{ nm}$) for numerical modeling in the main text (see Figs.2-10 and Table S1) not only because they correspond to realistic experimental situation, but primary because our parametric study presented in this Appendix predicts unusual hysteresis properties of $\langle P_3 \rangle$ and $\sigma_g$ for these values.

The top oxide thickness $h_o$ can vary in the experimental range (5 – 50) nm and $h_{DL}$ usually does not exceed several nm. To study the parameters $h_o$ and $h_{DL}$ effect on the hysteresis loops we performed numerical calculations, which results are shown in Figs.S4. Figure S4(a) shows how the charge concentration in graphene changes with $h_o$ increase at fixed other parameters. One can see from the figure the expected and trivial result – the loops become much slimmer with $h_o$ increase from 8 nm to 25 nm at fixed $\varepsilon_o = 80$. Figure S4(b) shows how the charge concentration in graphene changes with the increase of $h_{DL}$ from 0.5 nm to 2.5 nm at fixed other parameters. From the figure the expected result is that the loops looses their ferroelectric-like shape, becomes horizontally tilted and elliptic-like with $h_{DL}$ increase from 0.5 nm to 2.5 nm at fixed $\varepsilon_{DL} = 80$. 

25
Figure S1. Dependences of polarization $\langle P_3 \rangle$ on the reduced thickness $h_o/\varepsilon_o$, calculated for fixed values of $h_{DL}/\varepsilon_{DL} = 0.002, 0.005, 0.01$ nm and 0.03 nm (see curves 1, 2, 3 and 4) under zero gate voltage $U = 0$ at $T=300, 350, 400$ and $450$ K (a, b, c and d). Other parameters are listed in Table S1.
Figure S2. Dependences of graphene charge $\sigma_G(\Phi)$ on the reduced thickness $h_O/\varepsilon_O$, calculated for fixed values of $h_{DL}/\varepsilon_{DL}=0.002, 0.005, 0.01$ nm and 0.03 nm (see curves 1, 2, 3 and 4) under zero gate voltage $U = 0$ at $T=300, 350, 400$ and 450 K (a, b, c and d). Other parameters are listed in Table S1.

Figure S3. Dependences of polarization $\langle P_3 \rangle$ (a) and graphene charge $\sigma_G(\Phi)$ (b) on the reduced thickness $h_O/\varepsilon_O$, calculated for fixed values of $h_{DL}/\varepsilon_{DL}=0.002, 0.005, 0.01$ nm (see curves 1, 2, 3).
under zero gate voltage \( U = 0 \) at \( T=500 \text{ K} \) (a, b, c and d). The spontaneous polarization \( \langle P_z \rangle \) and corresponding charge \( \sigma_c(\Phi) \) are absent for \( h_{DL}/\varepsilon_{DL} = 0.03 \text{ nm} \). Other parameters are listed in Table S1.

Figure S4. (a) The charge concentration in graphene changes with \( h_O \) increase from 8 nm to 25 nm at fixed \( \varepsilon_O = 20 \), temperature 300 K, gate voltage 5V, period = 1000s, \( \varepsilon_{DL} = 80 \) and \( h_{DL} = 0.4 \text{ nm} \). (b) The charge concentration in graphene changes with \( h_{DL} \) increase from 0.5 nm to 2.5 nm at fixed \( \varepsilon_{DL} = 80 \), temperature 300 K, gate voltage 5V, period = 1000s, \( \varepsilon_O = 20 \) and \( h_O = 8 \text{ nm} \).

References


7 Zhang, X.-W.; Xie, D.; Xu, J.-L.; Zhang, C.; Sun, Y.-L.; Zhao, Y.-F.; Li, X.; Li, X.-M.; Zhu, H.-W.; Chen, H.-M.; Chang, T.-C. Temperature-dependent electrical transport properties in graphene/Pb(Zr_{0.4}Ti_{0.6})O_{3} field effect transistors. *Carbon* 2015, 93, 384–392.


18 Xia Hong, Emerging ferroelectric transistors with nanoscale channel materials: the possibilities, the limitations, Journal of Physics: Condensed Matter 28 (10), 103003 (2016)


Maksym V. Strikha and Anna N. Morozovska. Limits for the graphene on ferroelectric domain wall p-n-junction rectifier for different regimes of current. J. Appl. Phys. 120, 214101 (2016)

41 Ke Zou, Xia Hong, Derek Keefer, and Jun Zhu, Deposition of high-quality HfO2 on graphene and the effect of remote oxide phonon scattering, Physical review letters 105(12), 126601 (2010).